# Accelerating solitons for sliding-frequency filter systems

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(Received 8 November 2001; published 24 June 2002)

The sliding-frequency filter equation is shown to have similarity solutions which travel with steady profile but with *constant acceleration*. Over a wide range of the gain, filter strength and sliding-rate parameters, the pulse envelope is very well approximated by a sech profile. However, when the sliding rate is large, the chirp differs greatly from the usually assumed linear variation of frequency through the pulse. The amplitude and chirp are found for small and moderate sliding rate by a perturbation analysis and, for larger sliding rates, by solving a nonlinear eigenvalue problem for a nonautonomous differential equation.

DOI: 10.1103/PhysRevE.65.066615 PACS number(s): 42.65.Tg, 42.65.Wi, 42.81.Dp

## I. INTRODUCTION

Ultralong distance fiber-optic transmission using solitons has been demonstrated [1] using periodically spaced erbium-doped amplifiers to counterbalance fiber loss. To suppress amplification of unwanted noise, filters are introduced at each amplifier, allowing passage of signals only in a narrow band centered at the central frequency of the soliton. Even this allows degradation by noise, unless the soliton frequency is incrementally shifted at each amplifier. The soliton is able gradually to adapt to this frequency shift, but low-amplitude noise, being governed by linear equations, cannot. This is the principle of sliding-frequency filter (SFF) transmission [2]. In the limit for which amplifier spacing is short compared to the dispersion length, an appropriate evolution equation [3] is

$$\mathcal{N}u \equiv i u_Z + \tfrac{1}{2} u_{TT} + |u|^2 u = i\delta u + i\beta(\partial_T + i\hat{\alpha}Z)^2 u, \quad (1)$$

where $\delta$ is the (averaged) excess gain, $\beta$ is the filter strength, $-\hat{\alpha}$ is the (constant) frequency-sliding rate, $Z$ is the (scaled) propagation distance, and $T$ a retarded time.

It is widely known that the undamped nonlinear Schrödinger (NLS) equation $\mathcal{N}u=0$ possesses sech-envelope solutions at all carrier frequencies, viz.

$$u = A \exp i\{\Omega_1 T + \tfrac{1}{2}(A^2+\Omega_1^2)Z\} \operatorname{sech} A(T-\Omega_1 Z). \quad (2)$$

These show that the perturbation $\Omega_1$ in soliton speed is proportional to the shift in carrier frequency (with associated shift in wavelength being quadratic in both $\Omega_1$ and the soliton amplitude $A$).

When an SFF system is designed to transmit pulses with frequency shift $-\hat{\alpha}Z$ which is linear in the transmission "distance" $Z$, it is self-consistent to seek solutions to Eq. (1) having similarity variable

$$\zeta = T + aZ^2 + bZ. \quad (3)$$

These are "accelerating solutions"—recently shown to exist in a number of optical systems [4–6], such as dissipative media with nonlinear saturable gain and photorefractive beams and existing also [3] for a related integrable equation in which the right-hand side of Eq. (1) is replaced by $i\hat{\alpha}Zu$.

In Sec. II, the reduction of Eq. (1) to an ordinary differential Eq. (10) for a complex amplitude $W$, consistent with the similarity variable (3) is obtained. The "acceleration" is $a=\hat{\alpha}/2$ while, in the equation, $\delta$ and $\hat{\alpha}$ appear only in the combination $\hat{\alpha}/\delta^{3/2} \equiv \nu$ [see Eq. (10)]. In Sec. III, a perturbation method valid for any given $\beta$ but for $|\nu| \ll 1$ (i.e., $|\hat{\alpha}| \ll \delta^{3/2}$) yields a criterion determining the parameter $b$ and the initial conditions which allow Eq. (10) to possess isolated pulse solutions. Section IV extends analysis allowing estimation of the parameters and initial conditions in Eq. (10) which yield isolated pulse solutions for larger values of $\nu$ constrained only by the relation

$$27\nu^2\beta(1+4\beta^2) \lesssim 64.$$

The numerical and analytic results presented in Sec. V concerning Eq. (10) for general values of $\beta$ and for both small and large values of $|\nu|$ show that a profile of $|u|$ always exists which is remarkably close to a sech curve. The expressions for $b$ and for the pulse amplitude and width predicted in Sec. III are confirmed as good approximations for moderate values of $|\nu|$ (not just for $|\nu| \ll 1$). Also, the frequency within the pulse is found to have chirp which is *very well* approximated by a tanh curve, rather than the linear ramp usually assumed. The tanh dependence is predicted by the perturbation analysis of Sec. III, as is the relation between pulse half-width and amplitude.

## II. ACCELERATING SELF-SIMILAR SOLUTIONS

We seek a solution to Eq. (1) in the form

$$u(T,Z) = e^{i\theta(\zeta,Z)} F(\zeta) \quad (4)$$

with $\zeta$ given by Eq. (3) and with $a$, $b$, $\theta$, and $F$ real.

Insertion of Eq. (4) into Eq. (1) and multiplication by $2+4i\beta$ yields

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$$\begin{aligned}(1+4\beta^2)F''(\zeta)&+2\{2\beta[(\hat{\alpha}-2a)Z-b]\\&+i[(1+4\beta^2)\theta_\zeta+(2a+4\beta^2\hat{\alpha})Z+b]\}F'(\zeta)\\&-\{(1+4\beta^2)(\theta_\zeta)^2+2(2aZ+b+4\beta^2\hat{\alpha}Z)\theta_\zeta\\&+2\theta_Z+4\beta^2\hat{\alpha}^2Z^2-4\beta\delta-i[(1+4\beta^2)\theta_{\zeta\zeta}\\&+4\beta(\hat{\alpha}Z-2aZ-b)\theta_\zeta-4\beta\theta_Z+2\beta\hat{\alpha}^2Z^2-2\delta]\}F(\zeta)\\&+2(1+2i\beta)[F(\zeta)]^3=0.\end{aligned}\tag{5}$$

The ansatz (4) is self-consistent only if both the real and imaginary parts of the coefficients in Eq. (5) depend on $\zeta$ alone. From the coefficient of $F'(\zeta)$, we deduce immediately that $2a=\hat{\alpha}$ and that

$$\theta(\zeta,Z)=-\hat{\alpha}\zeta Z+\Theta(\zeta)+\Phi(Z),$$

so yielding the results $\theta_{\zeta\zeta}=\Theta''(\zeta)$, $\theta_\zeta+\hat{\alpha}Z=\Theta'(\zeta)$, and $\theta_Z=-\hat{\alpha}\zeta+\Phi'(Z)$. In order that the real part of the coefficient of $F(\zeta)$ is independent of $Z$, it is necessary that

$$\Phi'(Z)=\tfrac{1}{2}\hat{\alpha}^2Z^2+\hat{\alpha}bZ+c_0$$

for some constant $c_0$. This yields the expression

$$\theta(\zeta,Z)=\Theta(\zeta)-(\hat{\alpha}\zeta-c_0)Z+\tfrac{1}{6}\hat{\alpha}^2Z^3+\tfrac{1}{2}\hat{\alpha}bZ^2+c_1,\tag{6}$$

so that Eq. (5) reduces to

$$\begin{aligned}(1+4\beta^2)\{F''(\zeta)&+i[\Theta''(\zeta)F+2\Theta'(\zeta)F'(\zeta)]\\&-[\Theta'(\zeta)]^2F\}+2(1+2i\beta)\{ib[F'(\zeta)+i\Theta'(\zeta)F]\\&+(\hat{\alpha}\zeta-c_0-i\delta)F+F^3\}\\&=0.\end{aligned}\tag{7}$$

It is observed in [2] that Eq. (1), for $\hat{\alpha}\equiv0$, possesses special solutions in which

$$F=A\operatorname{sech}T,\qquad \theta=KZ+C\ln(\operatorname{sech}T),$$

which correspond to $b=0$, $c_0=K$. However, since these solutions impose four relations upon $A$, $C$, $K$, $\beta$, and $\delta$, they apply only for special combinations of $\beta$ and $\delta$. Fortunately, there is a generalization

$$F=A\operatorname{sech}\kappa(\zeta-\zeta_0),\qquad \Theta=C\ln[\operatorname{sech}\kappa(\zeta-\zeta_0)],\tag{8}$$

which satisfies Eq. (7) for $\hat{\alpha}=0$, $b=0$ provided that

$$\begin{aligned}C&=\frac{8\beta}{3+\sqrt{9+32\beta^2}}=\frac{\sqrt{9+32\beta^2}-3}{4\beta},\\A^2&=\frac{3\delta}{4\beta}(1+\sqrt{9+32\beta^2}),\\\kappa^2&=\frac{8\beta\delta}{3+8\beta^2-\sqrt{9+32\beta^2}},\qquad c_0=\frac{\delta}{2\beta}\sqrt{9+32\beta^2}.\end{aligned}\tag{9}$$

This defines a solution $F(\zeta)$ and $\theta(\zeta,Z)$ to Eq. (7) with $\hat{\alpha}=0$, $b=0$ for *all* choices of $\delta$ and $\beta$, with $\zeta_0$ arbitrary.

Observe that expressions (8) become singular as $\beta\to0$. Since they describe isolated pulses in which $\hat{\alpha}=0$, $b=0$, it is natural to rearrange Eq. (7) as

$$\begin{aligned}(1+4\beta^2)W''(Y)&+2(1+2i\beta)\{\mu_0-i+|W|^2\}W\\&=-2(1+2i\beta)\{iBW'(Y)+\nu YW\},\end{aligned}\tag{10}$$

where

$$\mu_0\equiv\frac{\hat{\alpha}\zeta_0-c_0}{\delta},\qquad Y\equiv\delta^{1/2}(\zeta-\zeta_0),$$

$$B\equiv\delta^{-1/2}b,\qquad \nu\equiv\delta^{-3/2}\hat{\alpha},$$

and

$$F(\zeta)e^{i\Theta(\zeta)}\equiv\delta^{1/2}W(Y).\tag{11}$$

## III. PERTURBATION ANALYSIS

Solutions to Eq. (10) may be sought by expanding in $\nu$ as

$$W(Y)=\hat{W}(Y)+\nu V(Y)+\cdots,\qquad B=\nu B_1+\cdots,\tag{12}$$

with $\hat{W}(Y)=\hat{A}\operatorname{sech}\hat{\kappa}Ye^{i\hat{\Theta}(Y)}$, $\hat{\Theta}(Y)=C\ln(\operatorname{sech}\hat{\kappa}Y)$, with $\hat{\kappa}=\delta^{-1/2}\kappa$, $\hat{A}=\delta^{-1/2}A$, and with $A$, $C$, and $\kappa$ given by Eq. (9), provided that $\beta$ is not too small. Then, inserting into Eq. (10) and taking $O(\nu)$ terms gives the linear differential equation for *complex* $V(Y)$:

$$\begin{aligned}\mathcal{L}V\equiv(1+4\beta^2)V''(Y)&+2(1+2i\beta)\\&\times\{(\mu_0-i)V+\hat{A}^2(\operatorname{sech}^2\hat{\kappa}Y)(2V+e^{2i\hat{\Theta}}V^*)\}\\&=-2(1+2i\beta)\{\hat{\kappa}B_1(C-i)\tanh\hat{\kappa}Y+Y\}\hat{A}\operatorname{sech}\hat{\kappa}Y.\end{aligned}\tag{13}$$

For Eq. (13), the four-dimensional space of complementary functions has a two-dimensional subspace composed of functions *even* in $Y$ and a two-dimensional subspace of functions *odd* in $Y$. Moreover, $i\hat{W}(Y)$ belongs to the first subspace and $\hat{W}'(Y)$ belongs to the second. Since, as $Y\to\infty$ and as $Y\to-\infty$ there exist two-parameter families of unbounded solutions, the *only* bounded contributions to the complementary function have the form $ic_1\hat{W}(Y)+c_2\hat{W}'(Y)$, with $c_1$, $c_2$ real.

Since the right-hand side of Eq. (13) is an odd function of $Y$, it is sufficient to seek odd, bounded solutions $V(Y)$ [adding to $V$ the even contribution $ic_1\hat{W}(Y)$ corresponds merely to a constant perturbation $\nu c_1$ to the phase $\hat{\Theta}(Y)$]. Moreover, it is permissible to set $V'(0)=id_1$ ($d_1$ real), since inclusion

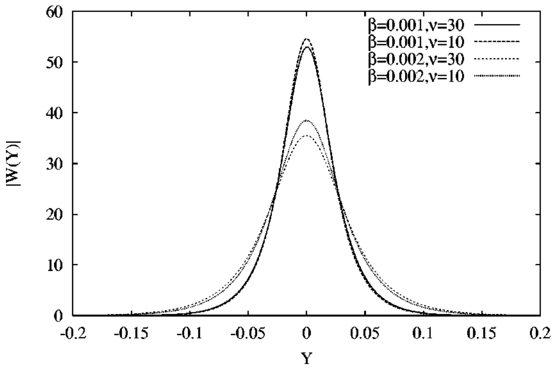


FIG. 1. Pulse envelopes $|W(Y)|$ governed by Eq. (10) for $\beta=0.001$ and 0.002, and for $\nu=10$ and $\nu=30$.

of $c_2\hat{W}'(Y)$ in $V(Y)$ merely corresponds to a perturbation in $\zeta_0$. Thus, by solving numerically the three initial-value problems

$$\mathcal{L}V_1=0,\quad V_1(0)=0,\quad V_1'(0)=i,$$

$$\mathcal{L}V_2=-2(1+2i\beta)\kappa\hat{A}(C-i)\tanh\hat{\kappa}Y\,\mathrm{sech}\,\hat{\kappa}Y,$$

$$V_2(0)=V_2'(0)=0,$$

$$\mathcal{L}V_3=-2(1+2i\beta)\hat{A}Y\,\mathrm{sech}\,\hat{\kappa}Y,\quad V_3(0)=V_3'(0)=0,$$

to yield unbounded, odd functions, it is possible to select $d_1$ and $B_1$ so that the combination

$$V(Y)=d_1V_1(Y)+B_1V_2(Y)+V_3(Y)$$

remains bounded. The corresponding approximation $W\approx\hat{W}(Y)+\nu V(Y)$ then has $W(0)=\hat{A}+O(\nu^2)$, $W'(0)=i\nu d_1+O(\nu^2)$, so that, correct to $O(\nu)$, the maximum of $|W|$ occurs at $Y=0$.

These calculations show that, for specified excess gain $\delta$, there exist isolated pulses which accelerate uniformly (with $a=\frac{1}{2}\hat{\alpha}$) and for which $|W(Y)|$ is very close to the familiar sech profile $\hat{A}\,\mathrm{sech}\,\hat{\kappa}Y=\hat{A}\,\mathrm{sech}\,\kappa(\zeta-\zeta_0)$. In these, the phase has *chirp* given by $\Theta'(\zeta)\approx-\kappa C\tanh\hat{\kappa}Y=-\kappa C\tanh\kappa(\zeta-\zeta_0)$ and the parameter $b$ in Eq. (3) is given by $b\approx\nu\delta^{1/2}B_1=\hat{\alpha}\delta^{-1}B_1$. These profiles and chirp are found (see Figs. 1 and 2) to give good approximations for a wide range of values of $\beta$ not only for small sliding rate $\nu\equiv\hat{\alpha}/\delta^{3/2}\ll 1$, but also for $\nu=O(1)$. Moreover, the predicted value of $b$ gives a very useful approximation for the parameter $B$ required in the computations in Sec. IV.

## IV. GENERAL TREATMENT

Equation (10) may be put into the canonical form

$$w''(y)+2(1+2i\beta)\{i\overline{B}w'(y)+(y-i\Delta+|w|^2)w\}=0, \tag{14}$$

through use of the substitutions

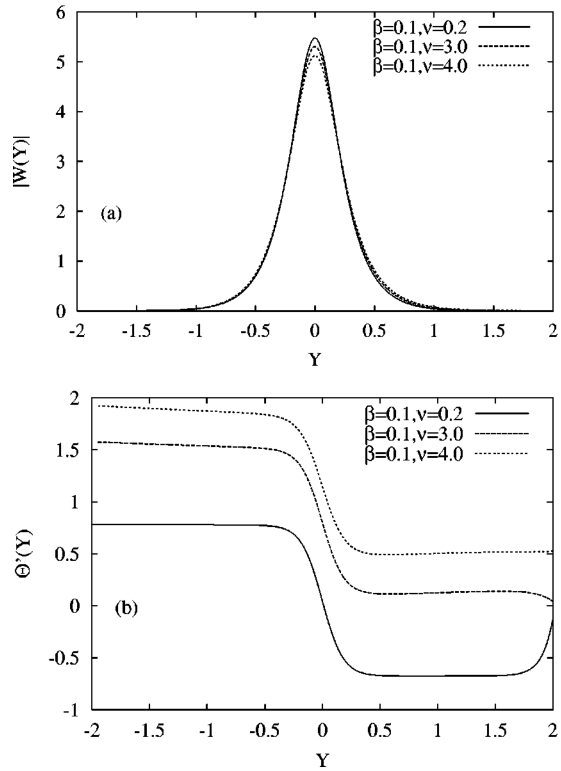


FIG. 2. (a) Some pulse envelopes $|W(Y)|$ for $\beta=0.1$; (b) the corresponding chirp $\Theta'(Y)$.

$$W(Y)=\nu^{1/3}(1+4\beta^2)^{1/6}w(y),$$

$$\nu Y+\mu_0=\nu^{2/3}(1+4\beta^2)^{1/3}y\equiv y/\Delta,$$

$$\overline{B}=\frac{B}{\nu^{1/3}(1+4\beta^2)^{2/3}},\quad \Delta=\frac{1}{\nu^{2/3}(1+4\beta^2)^{1/3}}. \tag{15}$$

Here, $\beta$ and $\Delta$ (or $\nu$) are parameters determined by the original Eq. (1), while $\overline{B}$ is adjustable. Our goal is to find pulse-like solutions to Eq. (14) for which $w\to 0$, $w'\to 0$ as $y\to\pm\infty$. For specified $\beta$ and $\Delta$, solutions may be expected to exist only for selected values of $\overline{B}$, with corresponding peak amplitude $|w|=|w|_{\max}$ at some location $y=y_0\equiv\Delta\mu_0$. Then, the corresponding parameters $\nu$, $B$, $\delta$, and $b$ follow from Eq. (11) when $\hat{\alpha}$ is specified, while the pulse is centered on the path

$$T+\tfrac{1}{2}\hat{\alpha}Z+\delta^2BZ^2=\zeta=(c_0/\hat{\alpha})+(1+4\beta^2)^{1/3}\nu^{-1/3}\delta^{-1/2}y_0$$

$$\equiv\zeta_0.$$

Since $c_0$ and $c_1$ correspond merely to a shift in reference phase and in the $T$ origin, they are set to zero without loss of generality.

In seeking solutions with $|w|$ small except near $\zeta=\zeta_0$, it is appropriate to analyze the linearization

$$w'(y)=v(y),$$

$$v'=-2(1+2i\beta)(y-i\Delta)w-2i\overline{B}(1+2i\beta)v, \tag{16}$$

which may be put into the matrix form $(w' \;\; v')^T = A(y)(w \;\; v)^T$, where the eigenvalues of $A$ are

$$\lambda \equiv \alpha + i\gamma = -i(1+2i\beta) \times\{\bar{B} \pm \sqrt{\bar{B}^2 + 2(1+2i\beta)^{-1}(y-i\Delta)}\}. \tag{17}$$

The real parts $\alpha_\pm$ and imaginary parts $\gamma_\pm$ of these two eigenvalues of $A$ are found from the identity

$$(\alpha - 2\bar{B}\beta) + i(\gamma + \bar{B}) = \mp i(1+2i\beta) \times \sqrt{\bar{B}^2 + 2(1+2i\beta)^{-1}(y-i\Delta)},$$

so yielding the compact representation

$$\lambda = \lambda_\pm = \alpha_\pm + i\gamma_\pm = -i\bar{B}(1+2i\beta) \pm \frac{\rho(y) + |\rho(y)|}{\{\mathrm{Re}\,\rho(y) + |\rho(y)|\}^{1/2}}, \tag{18}$$

where

$$\rho(y) \equiv -(1+2i\beta)\{y + \tfrac{1}{2}\bar{B}^2 + i(\beta\bar{B}^2 - \Delta)\}.$$

Since $\mathrm{Re}\,\rho(y) + |\rho(y)| > 0$ for all $y$, the eigenvalues $\lambda_\pm$ satisfy $\mathrm{Re}\,\lambda_+ \equiv \alpha_+ > \alpha_- \equiv \mathrm{Re}\,\lambda_-$. Moreover, except in some vicinity of the location $y = \bar{y} \equiv -\frac{1}{2}\bar{B}^2 + 2\beta(\beta\bar{B}^2 - \Delta)$ at which $\mathrm{Re}\,\rho(\bar{y}) = 0$, the eigenvalues have $\alpha_+ > 0$ and $\alpha_- < 0$. Thus, outside this restricted portion of the $y$ axis, there exists an *unstable manifold* of solutions which, near $|w| = 0$, has $(w' \;\; v')^T \approx \lambda_+(w \;\; v)^T$. Along these solutions, $|w|$ *decreases* as $y$ decreases, with $v \approx \lambda_+ w$. Also, there is a *stable manifold* on which $v \approx \lambda_- w$ (and $|w|$ decreases as $y$ increases). Each of these manifolds is two dimensional [since the system (16) is invariant under all mappings $\{w, v\} \mapsto \{we^{is}, ve^{is}\}$ for $s$ real]. For chosen $\{\Delta, \beta\}$, the required pulselike solution to Eq. (14) is described by a *connection* between these two manifolds. However, since $\mathrm{Re}\,\rho = -y - \frac{1}{2}\bar{B}^2 + 2\beta(\beta\bar{B}^2 - \Delta)$, it is clear that $|\alpha_+|$ and $|\alpha_-|$ remain small for $y > \bar{y}$, but grow as $y$ decreases below the value $\bar{y}$. This suggests that the pulse center $y = y_0$ lies in $y < \bar{y}$.

To confirm this conjecture, we consider the limiting equation ($\beta \to 0, \;\; \Delta \to 0$)

$$w''(y) + 2i\bar{B}w'(y) + 2yw + 2|w|^2 w = 0$$

and make the substitution $w = e^{-i\bar{B}y}\chi(y)$ to yield

$$\chi''(y) - \Gamma^2\chi + 2|\chi|^2\chi = -(\Gamma^2 + \bar{B}^2 + 2y)\chi. \tag{19}$$

When the right-hand side is neglected, Eq. (19) is the NLS equation having the familiar pulse solutions [cf. Eq. (2)]

$$\chi = \Gamma\,\mathrm{sech}\,\Gamma(y - y_0),$$

with center at $y = y_0$, with amplitude $\Gamma$, and with half-width $\Gamma^{-1}$. Neglect of the right-hand side is a good approximation provided that

$$|\Gamma^2 + \bar{B}^2 + 2y| \ll \Gamma^2 \quad \text{for} \quad \Gamma|y - y_0| < 4,$$

which is possible provided that $y_0 \approx -\frac{1}{2}(\bar{B}^2 + \Gamma^2)$, with $\Gamma^3 \gg 8$.

For chosen $\{\Delta, \beta\}$, a search procedure for $y_0$ and $\bar{B}$ is used, starting from the values $y_1$ and $\bar{B}_1$ (see Appendix). Then, motivated by the approximation $\chi \approx \Gamma_1\,\mathrm{sech}\,\Gamma_1(y - y_1)$, where $\Gamma_1 \equiv \{-\bar{B}_1^2 - 2y_1\}^{1/2}$, a location at which $|\chi| \approx \varepsilon$ is estimated as $y_- = y_1 - \Gamma_1^{-1}\ln(2\Gamma_1/\varepsilon)$, for some chosen $\varepsilon \ll 1$. Equation (14) is then integrated numerically from $y_-$ with initial conditions $w(y_-) = \varepsilon, \;\; w'(y_-) = \varepsilon\lambda_+(y_-)$ (and with $\beta$ and $\Delta$ moderately small) so yielding a maximum of $|w|$ near to $y = y_1$, followed by a minimum of $|w|$ near to $y = 2y_1 - y_-$. The parameters $y_-$ and $\bar{B}$ are then adjusted to reduce to $o(\varepsilon^2)$ the minimum of $|w|^2 + K|w'|^2$ occurring on a solution curve $w(y)$ for $y > y_-$ (with $K \approx |\lambda_-|^{-2}$). Once $y_-$ and $\bar{B}$ are identified yielding an acceptably small value for this minimum, the corresponding pulse center $y_0$ is determined by locating the maximum of $|w|^2$.

## V. NUMERICAL RESULTS AND DISCUSSION

For small $\beta$ and $\delta$ (large $\nu$), Eq. (A5) shows the significance of the parameter $\Delta\beta^{-1/3}$. Indeed, the initial approximation $\bar{B}_1$ exists only for $\Delta\beta^{-1/3} \geq \frac{3}{4}$, which corresponds to the threshold value for excess gain $\delta$ as a function of frequency sliding-rate $\hat{\alpha}$ predicted by Kodama and Wabnitz [7] (namely, $\hat{\alpha}^2\beta(1+4\beta^2) \leq \frac{64}{27}\delta^3$). Figure 1 shows some pulses $|W(Y)| = \delta^{-1/2}|u|$ for $\beta = 0.001$ and $\beta = 0.002$. For these values of $\nu$ $(= 10, 30)$, the associated frequency perturbations $\Theta'(Y)$ are almost constant, though with a small-amplitude tanh profile of chirp [even though $\nu$ is not small, c.f. Eq. (8) and Sec. III]. Other calculations for $\beta = 0.001$ confirm the existence of similar pulses for $\Delta\beta^{-1/3} = 0.75$, but show that the boundary for the nonexistence of pulses is remarkably close to $\Delta = \frac{3}{4}\beta^{1/3}$ throughout $\beta < 0.04$. In fact, when pulses exist, two families exist [corresponding to the two negative roots of Eq. (A5)]. Pulses of the second family have smaller peak amplitude, but are unstable according to variational theory [7,8].

For larger filter strength $\beta = 0.1$ and smaller $\nu = \hat{\alpha}/\delta^{3/2}$, similar narrow pulses close in shape to a sech profile exist (see Fig. 2). As $\nu$ increases for fixed filter strength $\beta$, the peak value of $|W|$ decreases gradually and the pulse broadens. The pulses become more noticeably asymmetric for larger $\nu$ and the tanh-profile chirp $\Theta'(Y)$ $[= \delta^{-1/2}(\theta_\zeta + \hat{\alpha}Z)]$ becomes prominent [see Fig. 2(b)]. Indeed, expressions (8) remain remarkably accurate even for $\nu = 3.0$, so that the perturbation analysis of Sec. III is a useful description of

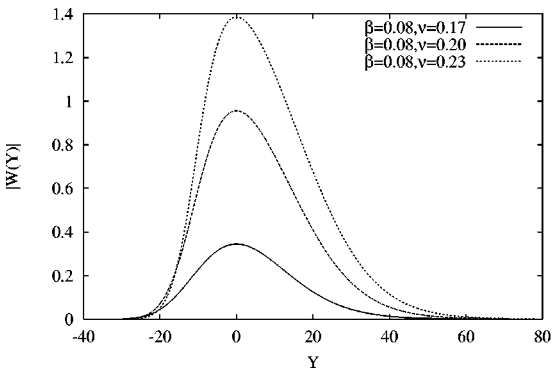


FIG. 3. $|W(Y)|$ for broad, asymmetric pulses existing for $\beta = 0.08$ when $\nu$ takes moderate values.

these pulses far beyond its expected region of validity $\nu \ll 1$.

However, for $\beta \approx 0.08$ and $\nu \approx 0.20$ a second family of smaller amplitude decidedly asymmetric, broad pulses have been found by numerical search, as shown in Fig. 3. These pulses, with moderate filter strength and with sliding-rate $\hat{\alpha}$ and excess gain $\delta$ comparably important, have parameter $b$ of much larger modulus than the narrow pulses. Also, the chirp $\Theta'(Y)$ for these pulses is much closer to the commonly assumed linear ramp than to the tanh profiles shown in Fig. 2(b). Moreover, a stability analysis currently being undertaken shows that these pulses are stable in this parameter range.

In summary, for a remarkably wide range of the parameters $\beta$, $\delta$, and $\hat{\alpha}$, there are pulses which are exactly nondistorting and which accelerate uniformly. Typically, their profile is remarkably close to a sech curve, even though the frequency increases linearly with $Z$ and has a pronounced chirp through the pulse.

## ACKNOWLEDGMENTS

Ch.R. would like to thank the Department of Science and Technology, Government of India, for financial support. M.F. acknowledges support from the Sub-Programa Ciência e Tecnologia do 2° Quadro Comunitário de Apoio (Grant No. PRAXIS XXI/BD/21642/99).

## APPENDIX

By introducing into Eq. (14) the substitutions $w = e^{-i\bar{B}y}\chi(\eta)$, $\eta = y - y_0$, we obtain

$$\chi'' + (2\chi\chi^* - \Gamma^2)\chi = F(\chi,\chi^*,\chi',\eta), \tag{A1}$$

where

$$F(\chi,\chi^*,\chi',\eta) = 4\beta\bar{B}\chi' - 4i\beta\bar{B}^2\chi - 4i\beta\chi^2\chi^* - 2\{\eta + 2\beta\Delta + 2i\beta(y_0+\eta) - i\Delta\}\chi \tag{A2}$$

and where * denotes a complex conjugate.

From Eq. (A1) and its complex conjugate, we find the two identities

$$\frac{d}{d\eta}(\chi^{*\prime}\chi' + \chi^2\chi^{*2} - \Gamma^2\chi\chi^*) = \chi^{*\prime}F + \chi' F^*,$$

$$\frac{d}{d\eta}(\chi^*\chi' - \chi\chi^{*\prime}) = \chi^*\chi'' - \chi\chi^{*\prime\prime} = \chi^* F - \chi F^*.$$

Hence, for pulses with $|\chi| \to 0$ as $\eta \to \pm\infty$, two integral conditions arise

$$\int_{-\infty}^{\infty}(\chi^{*\prime}F + \chi' F^*)d\eta = 0, \qquad \int_{-\infty}^{\infty}(\chi^* F - \chi F^*)d\eta = 0.$$

Introducing into these the representations $\chi = \rho e^{i\phi}$, $\chi^* = \rho e^{-i\phi}$ yields

$$\int_{-\infty}^{\infty}\rho^2 d\eta + 4\beta\bar{B}\int_{-\infty}^{\infty}(\rho')^2 d\eta - 2(2\beta\bar{B}^2 - \Delta + 2\beta y_0)\int_{-\infty}^{\infty}\rho^2\phi' d\eta + 4\beta\bar{B}\int_{-\infty}^{\infty}\rho^2(\phi')^2 d\eta - 4\beta\int_{-\infty}^{\infty}\eta\rho^2\phi' d\eta - 4\beta\int_{-\infty}^{\infty}\rho^4\phi' d\eta = 0 \tag{A3}$$

and

$$(2\beta\bar{B}^2 + 2\beta y_0 - \Delta)\int_{-\infty}^{\infty}\rho^2 d\eta + 2\beta\int_{-\infty}^{\infty}\rho^4 d\eta + 2\beta\int_{-\infty}^{\infty}\eta\rho^2 d\eta - 2\beta\bar{B}\int_{-\infty}^{\infty}\rho^2\phi' d\eta = 0. \tag{A4}$$

To leading order in $\beta$ and $\Delta$, we have $\rho = \Gamma_1 \operatorname{sech}\Gamma_1\eta$ with $\phi' = O(\beta,\Delta)$, so that by approximating $\bar{B}$ and $y_0$ by $\bar{B}_1$ and by $y_1 = -\frac{1}{2}(\Gamma_1^2 + \bar{B}_1^2)$ we obtain the leading-order approximations

$$\Gamma_1^2\int_{-\infty}^{\infty}\operatorname{sech}^2\Gamma_1\eta\, d\eta + 4\beta\bar{B}_1\Gamma_1^4\int_{-\infty}^{\infty}\operatorname{sech}^2\Gamma_1\eta\tanh^2\Gamma_1\eta\, d\eta = 0$$

and

$$(2\beta\bar{B}_1^2 + 2\beta y_1 - \Delta)\Gamma_1^2\int_{-\infty}^{\infty}\operatorname{sech}^2\Gamma_1\eta\, d\eta + 2\beta\Gamma_1^4\int_{-\infty}^{\infty}\operatorname{sech}^4\Gamma_1\eta\, d\eta = 0.$$

These yield the equations

$$\tfrac{4}{3}\overline{B}_1\Gamma_1^2\beta+1=0, \quad \overline{B}_1^2+\tfrac{1}{3}\Gamma_1^2=\beta^{-1}\Delta,$$

which are just the equations for the equilibrium points of the Kodama and Wabnitz [7] analysis. The resulting cubic equation for $\overline{B}_1$,

$$4\beta\overline{B}_1^3-4\Delta\overline{B}_1-1=0, \tag{A5}$$

has (two) roots with $\beta\overline{B}_1<0$ only for $\Delta\beta^{-1/3}\geqslant\frac{3}{4}$, then giving $y_1=(4\delta-\overline{B}_1^{-1})/8\beta$ and $\Gamma_1=(-4\beta\overline{B}_1/3)^{-1/2}$.


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